TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

21st YEAR OF QUALITY EDUCATION

CODE:- AG-TS-5-7989   REGNO:-TMC -D/79/89/36

GENERAL INSTRUCTIONS :-

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A,B,C and D. Section – A comprises of 8 question of 1 mark each. Section – B comprises of 6 questions of 2 marks each. Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Sections – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.
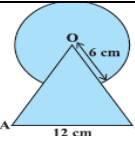
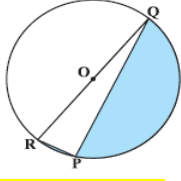### Pre-Board Examination 2012 -13

MATHEMATICS   CLASS X   (SA-2)
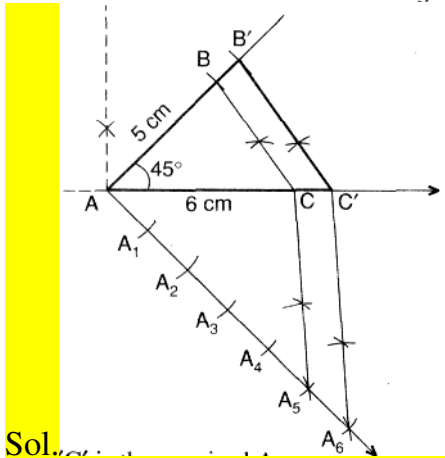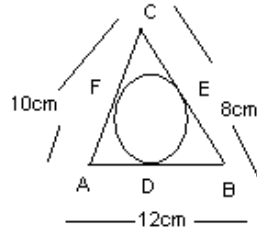
Time : 3 to $3\frac{1}{4}$ Hours   Maximum Marks : 90

SECTION A

Q.1	The distance of the point (- 4 , 5) from the X – axis is
	(a) - 4 units (b) 4 units (c) 5 units 9 d) -5 units Ans (c)

Q.2	A single letter is selected at random from the word '**PROBABILITY**' .The probability that it is a vowel is
	(a) $\frac{3}{11}$ (b) $\frac{4}{11}$ (c) $\frac{2}{11}$ (d) $\frac{5}{11}$ Ans (b)
Q.3	The points (-2,-5) , (2,-2) , (8,p) are collinear, then the value of p is (a) $-\frac{5}{2}$ (b) - $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{3}{2}$ Ans (c)
Q.4	9th term of an AP is 499 and 499th term is 9. the term which is equal to zero is (a) 507th (b) 508th (c) 509th (d) 510th Ans. B
Q.5	TP and TQ are two tangents to a circle with centre O, so that $\angle POQ = 120^0$, then $\angle OPT$ is equal to (a) 50^0 (b) 60^0 (c) 80^0 (d) 90^0 Ans d
Q.6	The quadratic equation whose roots are twice the roots of $2x^2 -5x+2=0$ is (a)$8x^2 -10x+2=0$ (b) $x^2 -4x+4=0$ (c) $x^2 -5x+4=0$ (d)$2x^2 -5x+2=0$ Ans. c
Q.7	The area of the shaded region in Fig. , where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre (a) $(36\sqrt{3})cm^2$ (b) $(660+36\sqrt{3})cm^2$ (c) $(\frac{660}{7}+36\sqrt{3})cm^2$ (d) None Ans c
Q.8	The coordinates of the middle points of the sides of a triangle are (4, 2) (3, 3) and (2, 2), then the coordinates of its centroid are (a) (3, 7/3) (b) (3, 3) (c) (4,3)(d) none of these (Ans. a)
	### SECTION B
Q.9	Form a quadratic equation with rational coefficients, one of whose roots is

Target Mathematics by- Agyat Gupta ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O)   Mobile : 9425109601; 9425110860;9425772164(P)

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$\dfrac{2-\sqrt{3}}{5}$. **Ans.** $s=\dfrac{4}{5}; p=\dfrac{1}{25}; 25x^2-20x+1=0$

Q.10	The sum of three numbers in A.P. is 27 and their product is 648. Find the numbers. Ans : 6 , 9 12

OR

If S_n denotes the sum of n terms of an AP whose common difference is **2** and 1st term is a. Find $S_n - 2S_{n-1} + S_{n-2}$ .Ans : $T_n - T_{n-1}$ = d

$T_n = S_n - S_{n-1}$
$T_{n-1} = S_{n-1} - S_{n-2}$
$S_n - 2 S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$
$\qquad = (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$
$\qquad = T_n - T_{n-1} = d$

Q.11	How many spherical lead shots each 4.2cm in diameter can be obtained from a rectangular solid of lead with dimensions 66cm, 42cm, and 21cm ? $(use\,\pi = 22/7)$ Ans 1500
Q.12	Find the value of x, if the distance between the points (x, -1) and (3,2) is 5. Ans -1 , 7 .
Q.13	Find the probability of getting a number between 1 and 100 which is divisible by 1 and itself only .Ans (25/98)
Q.14	Find the area of the shaded region in Fig. , if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Ans ($Area = 160cm^2$

SECTION C

Q.15	Find the values of x for which the distance between the points P (2, - 3) and Q (x, 5) is 10 units.Sol. PQ = 10 units...(Given) $\therefore$ PQ^2 = 100... (Squaring both sides) (x - 2)^2 + (5 + 3)^2 = 100(By distance formula) $\Rightarrow$(x-2)^2 + 64=100 $\Rightarrow$ (x - 2)^2=100-64 =36 $\Rightarrow$ x-2 = $\sqrt{36}$ = ±6 $\Rightarrow$ x-2 = 6orx-2=-6 $\therefore x = 8$ or $x = -4$

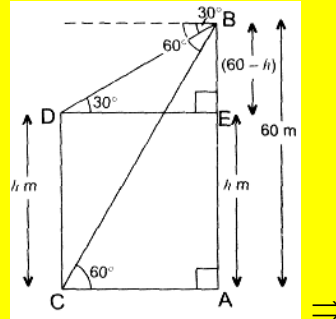| Q.16	Construct a ΔABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45°, then construct a triangle similar to the given triangle whose sides are $\frac{6}{5}$ of the corresponding sides of the ΔABC. Sol. ∴ ΔAB'C' is the required Δ.
Q.17	From your pocket money, you save Re. 1 on day 1, Rs. 2 on day 2, Rs. 3 on day 3 and so on. How much money will you save in the month of March 2008? Sol. Let total money saved be Rs. x $\therefore$ x = 1 +2 + 3 +.... + 31(31 days in March) $= \dfrac{31}{2}[1+31]$ $\left[\because S_n = \dfrac{n}{2}(a+l)\right.$ $= \dfrac{31}{2} \times 32 = 496$ 1st term, $a=1$ last term, $l=31$, $n=31$ Total money saved = Rs. 496
Q.18	If circle is inscribed in a ΔABC having sides 8cm, 10cm and 12cm as shown in figure. Find AD, BE and CF. Ans x =7,y=5,z=3
Q.19	Given that one root of the quadratic equation ax^2 + bx + c =0 is three times the other, show that 3 b^2 = 16 ac.

OR

Solve the quadratic equation $9x^2 - 15x + 6 = 0$ by the method of completing the square . **Ans.** 1 ; 2/3 .

$$x^2 - \frac{15x}{9} = \frac{-6}{9} \Rightarrow \left(\frac{5}{6}\right)^2 + x^2 - \frac{5x}{3} = \frac{-2}{3} + \left(\frac{5}{6}\right)^2$$

$$\left[x - \left(\frac{5}{6}\right)\right]^2 = \frac{1}{36} \Rightarrow x - \frac{5}{6} = \pm\frac{1}{6}$$

Q.20 Prove that the angle between two tangents drawn from an external points to a circle is supplementary to the angle subtended by the line segment joining the point of contact at the center .

Q.21 There are two poles, one each on either bank of a river, just opposite to each other. One pole is 60 m high. From the top of this pole, the angles of depression of the top and the foot of the other pole are 30° and 60° respectively. Find the width of the river and the height of the other pole. **Sol.** Let AB be the first pole and CD be the other one . CA is the river . Draw DE ⊥ AB . Let CD = h m = AE

BE = (60 - h) m . In rt. ΔBAC, $\frac{BA}{CA}$ =∴ $\frac{60}{CA} = \sqrt{3}$ tan 60°

CA= $\frac{60}{\sqrt{3}} \Rightarrow$ CA = $\frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$

∴ Width of river, CA = 20√3 m = 20(1.73) (∵ 3 = 1.73) = 34.6 m Now, In rt. ΔBED $\frac{BE}{DE}$ = tan 30° ∴ $\frac{60 - h}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$ ⇒ $\frac{60-h}{20} = 1$ ⇒60- h = 20 ⇒ h = 60-20 = 40 ∴ Height of the other pole = 40 m

Q.22 Three numbers are in the ratio 3: 7: 9. If 5 is subtracted from the second, the resulting numbers are in A.P. Find the original numbers. Ans Let nu. Is 3x , 7x 9x acc. To condition 2 (7x- 5) = 3x + 9x : x = 5 & numbers are 15 , 35 & 45 .

Q.23 OAQB is a quadrant of a circle with centre O. (see figure) C is mid point of OB. CD=CO= 7cm find the area of the shaded region.
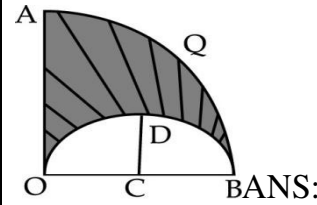
BANS:
CD = CO + 7cm
∴ OA = OB = 14cm
Area of shaded portion
= Area of quadrant OAQB − ar of semicircle ODB
= $\frac{1}{4}\pi(14)^2 - \frac{\pi}{2}(7)^2$
= $\frac{1}{4} \times \frac{22}{7} \times 14 \times 14 - \frac{22}{7} \times \frac{7 \times 7}{2}$
= 154 − 77 = 77cm²
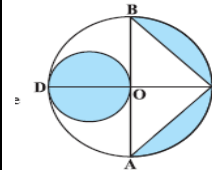
OR

In Fig., AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. Ans ($Area = 66.5 cm^2$

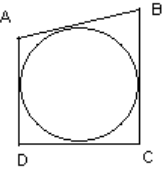**Q.24** Using A (4,-6), B(3,-2) and C(5,2),verify that a median of the triangle ABC divides it into two triangles of equal areas . Ans (each $Area = 3$ $unit^2$
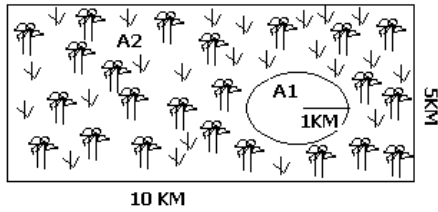
OR

The area of a triangle is 5. two of its vertices are (2,1) and (3,-2). The third vertex is (x,y) where y = x+3. find the co-ordinates of the third vertex. Ans $\left(\frac{7}{2}, \frac{13}{2}\right) or \left(-\frac{3}{2}, \frac{3}{2}\right)$

SECTION D

Q.25	If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$.
Q.26	Saraswati public school decides to award the prizes for punctuality , honesty, bravery and politeness (one for each) . The total amount of prizes is Rs. 2800 . If each prizes in order has the difference of Rs. 200 , find the amount decided for each prize . Which values are awarded doing so . ans : lrt the amount decided for the prizes are x , x – 200 , x – 400 , x – 600 respectively . sum of prizes = Rs. 2800 , x + x – 200 + x – 400 + x – 600 = 2800 then x = 1000 . prize for punctuality Rs.1000 , prize for honesty Rs. 800 , prize for bravery Rs. 600 , prize for politeness Rs. 400 . Here Saraswati Public School awarded for value of life i.e. Punctuality , Honesty , Bravery & Politeness . *There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.)*
Q.27	In the given fig., a circle touches all the four sides of a quadrilateral ABCD with sides AB = 6cm, BC = 7cm and CD = 4cm. Find AD. Ans 3cm
Q.28	From a solid cylinder whose height is 8cm. and radius 6cm, a conical cavity of height 8cm. and base radius 6cm. is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid. Ans VOLUME = 602 .88 Cubic Cm & S.A=602.88 sq.cm OR Water in a canal, 30 dm wide and 12 dm deep, is flowing with a speed of

10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed? Ans. 30 minutes

Q.29	Some students arranged a picnic. The budget for food was Rs. 240. Because four students of the group failed to go, the cost of food to each student got increased by Rs. 5. How many students went for the picnic? **Sol.** Let the number of students who arranged the picnic be x . Then the number of students who attended the picnic = *(x - 4)* .Total cost of food = Rs. 240 **A.T.Q.** $\dfrac{240}{x-4} - \dfrac{240}{x} = 5$ $\Rightarrow \dfrac{240[x-(x-4)]}{(x-4)x} =$ 5 $\Rightarrow \dfrac{240\times4}{(x-4)x} = 5$ $\Rightarrow \dfrac{48\times4}{(x-4)x} = 1 \Rightarrow$ (x-4)x=192 $\Rightarrow x^2$-4x- 192 = 0 $\Rightarrow$ x²-16x + 12x-192 = 0 $\Rightarrow$ x(x-16) + 12(x-16) = 0 $\Rightarrow$(x-16)(x + 12) = 0 $\Rightarrow$ x - 16 = 0 or *x* + 12 = 0 x = 16 or x = - 12 (Rejecting) ($\because$ Number of students can not be - ve) $\therefore$Number of students who actually went for the picnic = 16-4 = **12** OOR A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 250 km/hour from the usual speed. Find its usual speed. Sol. Let the usual speed of plane = x km/hour then the increased speed of the plane = (x + 250) km/hour Distance = 1500 km $\dfrac{1500}{x} - \dfrac{1500}{x+250} = \dfrac{1}{2}$ $\left[\because \text{Time} = \dfrac{\text{Distance}}{\text{Speed}},\ 30 \text{ mins.} = \dfrac{1}{2}\text{hr.}\right]$ **A.T.Q.** $\Rightarrow \dfrac{1500(x+250-x)}{x(x+250)} = \dfrac{1}{2}$ $\Rightarrow$x(x +250)=1500 × 250×2$\Rightarrow$ x²+250x -750000 = 0$\Rightarrow$ x² + 1000x - 750x - 750000 = 0 $\Rightarrow$x(x+ 1000)- 750(x+ 1000) = 0$\Rightarrow$(x+1000)(x -750) = 0 $\Rightarrow$ x+ 1000 = 0 or x -750 = 0 $\Rightarrow$ x = -1000 or x = 750 $\therefore$ (Speed of a plane can not be negative) $\therefore$x = 750 $\therefore$Usual speed of the plane = **750 km/hr**
Q.30	An army helicopter was flying over the forest , somehow it found missing . After few days , a news flashed on the TV news channel , that the helicopter got crashed in a rectangular region of the forest .

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(a) Find the chances of the helicopter crashed inside the circle . (b) Which area A_1 **or** A_2 has the maximum chance **of the** helicopter crash ? (c) How can a helicopter be useful in an area suffered by any natural calamity ? Discuss . (d) As the helicopter was flying over the forest , find the chances of the crash outside the rectangular region . ans : Total area of the rectangular region n (S) = $5 \times 10 = 50\ km^2$. Inside the rectangular region , a circular area is defined . Whose area , n (A_1) = πr^2 = 3.14×1^2 = $3.14\ km^2$. Now Area of the region remaining around the circle , n(A_2) = Total area of rectangle – area of circle = 50 – 3.14 = 46.86 sq km (a) Chances of the crash inside the circle , P(A_1) = area of circular region / area of rectangular region = $\dfrac{n(A_1)}{n(s)}$ = $\dfrac{3.14}{50}$ = .0628 (b) Chances of the crash inside the circle , P(A_2) = area of region remaining around the circle / area of rectangular region = $\dfrac{n(A_2)}{n(s)}$ = $\dfrac{46.86}{50}$ = .9372 . Hence clear that P(A_2) has more chance for the crash . So area A_2 will have more chances (c) Natural calamities like earthquake , flood or draught etc . , causes damages and harms the people . Helicopter is the medium to get those people's life rescued from such calamities . Aids can be as follows (i) providing food to the drowne areas. (ii) Providing first aid etc. (d) Since the helicopter was flying over the rectangular forest region . So , there is no chance of the crash outside the forest . The probability of the crash outside the forest is zero. (*There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.*).

Q.31	How many terms of the A.P. : -15, -13, -11, …… are needed to make the sum -55? Explain the reason for double answer? ANS: ½ AP : $-15, -13, -11, ...$ $a = -15,\ d = a_2 - a, = -13 - (-15) = 2$ Let $S_n = -55$ $\dfrac{n}{2}[2a + (n-1)d] = -55$ $n[2(-15) + (n-1)2] = -110$ $n[-30 + 2n - 2] = -110$ $n[-32 + 2n] = -110$ $2n^2 - 32n + 110 = 0$ $n^2 - 16n + 55 = 0$ $n^2 - 11n - 5n + 55 = 0$ $n^2 - 11n - 5n + 55 = 0$ $n(n-11) - 5(n-11) = 0$ $(n-5)\ (n-11) = 0$ $\Rightarrow n = 5\ \ or\ \ n = 11$ 1 $\therefore$ Sum of first 5 terms as well as first 11 terms is -55. because 6th term to 11th term cancel out i.e. their sum is zero. ½
Q.32	A Pen stand made of wood in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15cm by 10cm by 3.5cm. The diameter if each depression is 1cm and the depth is 1.4 cm. finds the volume of the wood in the entire stand.

Volume of cuboid $= 15 \times 10 \times 3.5 = 525$ cm³
Radius of each expression $= 0.5$ cm
Depth $= 1.4$ cm

Volume of 4 conical depression $= 4 \times \dfrac{1}{3} \times \dfrac{22}{7} \times (0.5)^2 \times 1.4$

$= 1.47$ cm³

ANS: Volume of wood $= 525 - 1.47 = 523.53$ cm³

Q.33	From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60°respectively, Find the height of the lamp post. ANS:
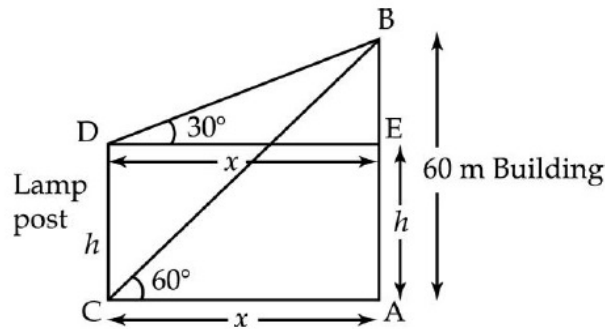
In rt ΔDEB, $\dfrac{x}{60-h} = \cot 30°$ In rt ΔCAB,

$\Rightarrow \dfrac{x}{60-h} = \sqrt{3}$ $\dfrac{x}{60} = \cot 60°$

$\Rightarrow x = (60-h)\sqrt{3}$ $\dfrac{x}{60} = \sqrt{3}$

$x = \dfrac{60}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$

$\Rightarrow (60-h)\sqrt{3} = 20\sqrt{3}$

$\Rightarrow 60-h = 20$

$h = 40$ m

Q.34	If the equation $\left(1+m^2\right)n^2 x^2 + 2mncx + \left(c^2 - a^2\right) = 0$ has equal roots of x, prove that $c^2 = a^2\left(1+m^2\right)$.

$(1+m^2)\, n^2.x^2 + 2mnc.x + (c^2 - a^2) = 0$ has equal roots, $\therefore$

$\{2mnc\}^2 = 4\{(1+m^2)\, n^2\}\{c^2 - a^2\}$

$4\, m^2 n^2 c^2 = 4\left(n^2\right)\left(1+m^2\right)\left(c^2 - a^2\right)$

$\Rightarrow m^2 c^2 = c^2 - a^2 + m^2 c^2 - m^2 a^2$ $\Rightarrow 0 = c^2 - a^2 - m^2 a^2$

$\boxed{\Rightarrow c^2 = a^2\,(1+m^2)}$

ANS:

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<u>**TO FOLLOW, WITHOUT HALT, ONE AIM :**</u>

THERE'S THE SECRET OF SUCCESS .

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